the dimensions of the sample and of the resonator, the resonator resonant frequency and Q-factor, as well as those resulting from assymetry of various kinds, is presented by the authors in [4], together with methods for a minimalization of these errors.

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High-Accuracy Wide-Range Measurement Method for Determination of Complex Permittivity in Reentrant Cavity: Part B—Experimental Analysis of Measurement Errors

ANDRZEJ KĄCZKOWSKI AND ANDRZEJ MILEWSKI

***Abstract*—In Part A, the measurement method of $\hat{\epsilon}$ was presented from the mathematical viewpoint. Experiments undertaken in this section were carried out in order to illustrate the theoretical thesis of Part A, as well as to verify the postulated calculation method and analyze measurement errors.**

I. MEASUREMENT APPARATUS

A. Resonator

IN THE EXPERIMENTAL part of this work, a resonator tunable by capacitance and inductance adjustment was used (Fig. 1) with functional dimensions: $r_1=14$ mm, $r_2=48$ mm, $L_1=32$–42 mm, $L=57$–500 mm, $d=0$–10 mm. As a result of the aforementioned adjustments, measurements at a wide range of frequencies 200–1000 MHz were possible.

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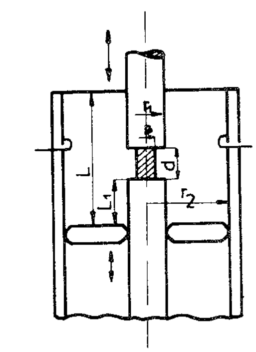

Fig. 1. Tunable reentrant cavity.

B. Q Factor and Resonant Frequency Measurement System

Measurements of the Q factor and resonant frequency were carried out by utilizing a Wobbulator, whose operating principle is shown in the block diagram (Fig. 2)

This apparatus makes it possible to obtain an accuracy of 1 percent on the Q-factor measurement, and 10^{-3}

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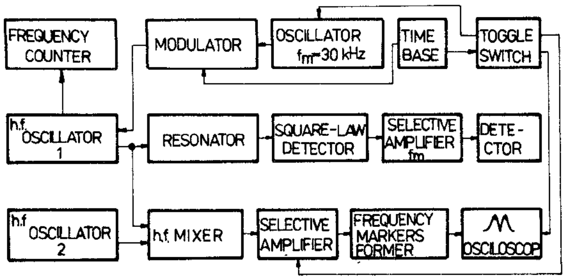

Fig. 2. Block diagram of the system for measuring the resonant frequency f and the Q factor.

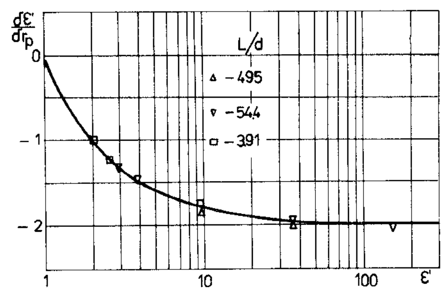

Fig. 3. Ratio of transformation error of the sample radius r_p on the permittivity error.

percent on the resonant frequency. The construction and principles of this system are elaborated in [2].

C. Materials

To test the method, samples of the following materials were used: polytetrafluoroethylene, quartz glass, alundum, ceramics N-47, rutile ceramics, and barium titanate ceramics E-300. The solids were chosen to represent a wide range of permittivity ($\epsilon'=2$–300) as well as various levels of dielectric losses. All samples had a cylindrical shape of 2–28-mm diameter and 0.6–10-mm length.

II. Measurement Errors Analysis

The sample permittivity $\hat{\epsilon}$ is a function of the following parameters: 1) dimensions of the resonator and of the sample: r_1, r_2, r_p, L, L_1, and d; and 2) resonant frequencies and Q factors of the resonator with and without the dielectric sample: f, Q, f_0, and Q_0, respectively.

Any measurement error of the above parameters has an effect on the final error of the sample permittivity. Moreover, facts not taken into account in the algorithms, whose influence can only be estimated experimentally, can have an effect on the accuracy of the $\hat{\epsilon}$ measurement. The differences between the real configuration of the resonator and its ideal mathematical model, the eccentric placement of the sample in the cavity and the possible presence of air gaps between the sample and the core of the cavity, all belong to the above facts.

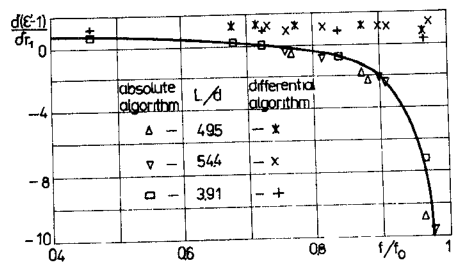

Fig. 4. Ratio of transformation error of the core radius r_1 on the electric susceptibility $(\epsilon'-1)$ error.

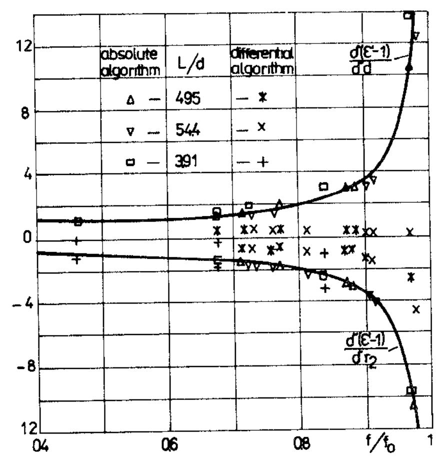

Fig. 5. Ratio of transformation error of the resonator radius r_2 and of the sample length d on the electric susceptibility $(\epsilon'-1)$ error.

A. The Influence of the Accuracy of Parameter on the Final Value of $\hat{\epsilon}$

The dependence of the sample permittivity $\hat{\epsilon}$ on the aforementioned values (r_p, r_1, r_2, L, L_1, d, f, f_0, Q, and Q_0) is so complicated, that the influence of the accuracy of the measurement of any partial variable cannot be evaluated by simple algebraic analysis. That is why the numerical analysis was applied for about 80 measurements. The cases were selected to cover a wide range: resonator properties, material permittivity, sample radii, and relative tuning of the resonant frequency by the sample.

The results of this analysis are illustrated in Figs. 3–6. To check which method of permittivity determination (absolute or differential) provides better accuracy, in Figs. 4 and 5, points marked ×∗× obtained from a differential algorithm analysis are plotted against points marked △O□

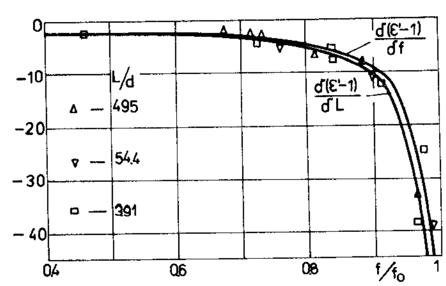

Fig. 6. Ratio of transformation error of the resonator length L and of the resonant frequency f on the electric susceptibility $(\epsilon'-1)$ error.

obtained from the analysis described in Part A. The former method (differential) is based on the determination of one of the resonator dimensions defined by the empty cavity resonant frequency f_0.

From these results the following conclusions can be drawn.

1) Measurement error of the sample radius r_p transfers into an error in describing permittivity, with the factor of proportionality $t(r_p) \leqslant -2$.

2) Measurement errors of the parameters r_1, r_2, L, d, and f transfer into an error of permittivity determination, and the greater the factor of the transformation (as far as the modulus is concerned) the smaller is the resonator frequency shift by the sample.

3) The factors transforming the errors r_1, r_2, and d into $\delta\epsilon'$ are much more suitable for the differential method, especially for small tuning of the resonator ($f:f_0 \rightarrow 1$).

As shown in many measurements, the effect of measurement error r_p, r_1, r_2, L, d, and f on the error of determination $\delta\epsilon''$ is negligible in comparison with the effects of the measurement errors of the Q factor. As in all resonators, so in this case, the change in the Q factor caused by the introduction of the sample into the resonator is decided by the transformation factor δQ on $\delta\epsilon''$.

B. Departures of the Real Resonator from the Accepted Mathematical Model

In this section, special attention is given to the following causes of measurement error, which were not analyzed in the previous section: 1) the effect of eccentric placement of the sample in the resonator; and 2) the effect of air gaps between the core of the cavity and the sample. It was decided to explore these problems experimentally.

During the investigation of $\hat{\epsilon}$ in the function of eccentric placement of the sample in the resonator, it was found that this effect is not large. Table I shows the errors $\delta\epsilon'$, resulting from many measurements.

Measurement errors of imaginary part of the permittivity were not able to be evaluated because differences were much smaller than the resolution of the apparatus ($\delta Q = 1$ percent).

It can be seen from the Table I that the error of eccentric placement of the sample is smaller as resonator length increases. Even in the worst case (the shortest cavity) this error, in practical measurements, is negligible without using additional instrumentation, because as seen during experiments, the eccentric placement of the sample in the resonator does not exceed 0.8 mm.

From a technical point of view, an analysis of the effects of air gaps poses a more difficult problem. The cause of this is the practical need to place the sample in the resonator without air gaps.

In order to evaluate the effective thickness of the air gap d_s, occurring between the sample and the resonator core, comparison measurement were made on ceramic samples, first without electrodes and then with electrodes on the frontal planes of the samples. These conducting electrodes were made by depositing silver-plated copper powder suspended in lacquer with a resistivity of $\rho = 8.8 \times 10^{-8}\ \Omega \cdot \text{m}$, after drying.

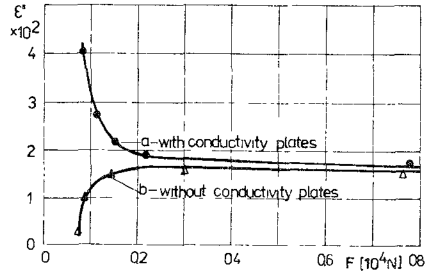

Fig. 7. Dependence of measuring value ϵ'' ceramics N-47 on force with which the cores are pressed on to the sample.

TABLE I
MEAN MEASUREMENT ERRORS OF PERMITTIVITY $\delta\epsilon'$(PERCENT) CAUSED BY ECCENTRIC PLACEMENT OF THIS SAMPLES ($r_p : r_1 \leqslant 0.2$) THE DISTANCE BETWEEN THE AXIS OF THE SAMPLE AND THAT OF THE RESONATOR $-a$, IS A MEASURE OF THE ECCENTRIC PLACEMENT OF THE SAMPLE

L [mm] \ a [mm]	2	5	10
56	-0,06	-0,35	-1,40
100	-0,03	-0,17	-0,67
150	-0,02	-0,09	-0,37
200	-0,01	-0,06	-0,24
250	0	-0,04	-0,16
300	0	-0,03	-0,12

TABLE II
THE EFFECT OF THE AIR GAP ON THE ϵ' MEASUREMENT OF A CERAMIC CYLINDER WITH A 5-mm DIAMETER

Name of material	d [mm]	ϵ_1'	ϵ_2'	d_s [mm]	$\frac{\epsilon_2'-\epsilon_1'}{\epsilon_2'}$ [%]
steatite ceramic	2	6,59	6,68	0,0046	1,3
	4	6,62	6,66	0,0045	0,6
	7	6,58	6,61	0,0053	0,4
N-47	2	34,11	37,82	0,0059	9,8
	4	34,85	37,97	0,0097	8,2
	7	36,97	37,89	0,0047	2,4
E-300	2	122,8	280,7	0,0092	56,3
	4	198,3	287,9	0,0063	31,1
	7	239,0	281,1	0,0044	15,0

where:

ϵ_1', ϵ_2' – the permittivity of the sample, without and with conducting electrodes, respectively.

During the experiments, phenomena were observed for which a mathematical formula is hard to find, but which have an important practical meaning.

1) It can be seen from the example results found in Table II that with an increase in the height of the samples, the effects of the air gap on the measurement ϵ' decreases, however for $\epsilon' > 10$ it is significant. The effective thickness of the gap, naturally, depends on the strength with which

the cores are pressed on to the sample. The results presented in Table II were obtained using small axis pressure $P=600$ N. With an increase of pressure in the range of 600–6000 N, the gap d_s decreased to about half its size.

2) An interesting dependence was obtained in investigating the imaginary part of permittivity of metalized and nonmetalized samples as a function of pressure. As an example of these measurements, one sample (of ceramic N-47) is presented in Fig. 7.

On the basis of the ϵ'' measurement as a function of pressure, it can be established that a more accurate measure of material losses can be obtained for nonmetalized samples due to the elimination of a significant portion of the contact resistivity (core-metalized layer) in the dissipation of energy. In general, with a metalized layer the ϵ'' results are overestimated; without it they are underestimated. In order to increase accuracy, it is necessary to use significant pressure P, which, however, must be smaller than the force which can destroy the sample. Associated with this is the need to use a hard material for the contact surface of the core, so that it will not be damaged by the hard ceramic sample.

III. Summary

In Part A, a theoretical analysis of the measurement method was present while in Part B, an experimental analysis of measurement error showed that the author's proposed measurement method for thin samples in a reentrant cavity allows for $\hat{\epsilon}$ measurements in a wide range of materials ($\epsilon'=2$–300, $\tan\delta=10^{-5}$–10^{-1}) with an adequate technical accuracy $\delta\epsilon'\leq 1$ percent, $\delta(\tan\delta)\leq 5$ percent $+5\times10^{-5}$.

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Wave Propagation through Weakly Anisotropic Straight and Curved Rectangular Dielectric Optical Guides

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***Abstract*—Wave propagation through weakly anisotropic straight and curved dielectric rectangular guides is studied using a coupled mode approach. The propagation constant, thus found, can be computed very easily if Marcatili's approximate field expressions for an isotropic guide are used. The result for the uniaxial case can then be extended for the biaxial crystal to the first order of approximation.**

I. Introduction

DIELECTRIC WAVEGUIDES of different shapes have found wide use in integrated optics and optical communication systems. The basic structures that have been studied extensively for optical applications are, in general, isotropic slabs, rectangular and circular cylindrical guides. However, in many diverse applications, such as dielectric cavity resonators (DCR), laser and masers, ESR spectroscopy, nonlinear optical devices, etc., the interaction may involve an anisotropic dielectric medium. Symmetrical [1], [2] as well as hybrid [3], [4] mode propagation in a uniaxial anisotropic rod have been extensively studied. The theory has since been extended to the weakly anisotropic case [5], biaxial rod [6], and hollow axially anisotropic structure [7] with possible applications in retinal receptor modeling, crystal-core guides, and gas laser resonating structures, respectively.

The work on planner structures, however, seems incomplete. The effect of anisotropy on a slab guide [8], [9] and mode coupling in general anisotropic guides [10] have been studied recently, but no reported result for rectangular guides is known to the present author. The difficulty in obtaining an exact analytical solution for the propagation constant for a simpler isotropic rectangular guide lies in matching boundary conditions everywhere. A lengthy numerical method [11] may be used, but an approximate solution [12], [13] offers better insight to the propagation and field behavior of the modes.

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